FREQUENCY OF THE THREE-PHASE R-C COUPLED OSCILLATOR PART II. INDUCTIVE ANODE LOAD RESISTANCE

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(*Received, December* 12, 1959)

ABSTRACT. When the three stages of a conventional three phase R–C oscillator are identical, the oscillations normally produced are of radio frequency $\omega = \sqrt{3}/CR$, where R and C are tuning resistance and capacitance. It was shown in Part I (Rakshit and Mallik, 1955) that this simple formula holds when the load resistance is non-reactive and the cathode impedance is zero or resistive. The case of inductive anode resistance when cathode impedance is (i) resistive and (ii) reactive has been discussed in the present paper. For resistive cathode impedance an inductance in series with anode resistance R causes an increase in frequency over the $\sqrt{3}/CR$ value up to a certain magnitude of the inductance but for still higher magnitudes the frequency goes on decreasing

The effect of reactive cathode when the anode load is inductive is the same as that for non-reactive anode load resistance discussed in Part I Inductive cathode impedance decreases the frequency of oscillation from the value for resistive cathode impedance while capacitive cathode impedance generally increases the same.

INTRODUCTION

It has been shown in Part I that the frequency of the three phase R-C oscillator is dependent on the cathode circuit impedance. The simple expression for frequency $\omega = \sqrt{3}/CR$ holds good, only when the cathode circuit impedance is zero or purely resistive and anode load resistance is non-reactive. When the cathode impedance is reactive the generated frequency will be greater or less than $\sqrt{3}/CR$ depending upon whether the cathode impedance is capacitive or inductive The effect of cathode impedance is prominent when the generated frequency is much below or much above the cathode resonant frequency.

In the present paper we shall discuss the condition when cathode impedance is zero, resistive, or reactive but anode load resistance is inductive. The addition of inductance with the load resistance increases the load impedance and makes it possible to maintain oscillation at comparatively low values of load resistance. In fact in our attempt to generate very high frequency it has been found that these inductances are essential for maintenance of oscillation (Rakshit and Mallik, 1953). Without the small inductance the oscillation frequency $\omega =$

$\sqrt{3}/CR$ and the phase shift per stage is 120 degrees. The presence of the small inductance causes a decrease in phase shift at this frequency and as a result the frequency of the maintained oscillation is increased. This increase in frequency with increasing inductance continues up to a certain maximum value and thereafter the frequency goes on decreasing and finally for still larger values of inductance the frequency becomes less than $\sqrt{3}/CR$ value.

THEORETICAL CONSIDERATION

In this case the anode load Z_0 consists of the tuning capacitor C shunted by the series combination of R and its associated inductance L.
Hence,

$$Z_0 = \frac{[R + j\omega\{L(1-\omega^2 LC) - CR^2\}]}{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2} \qquad \ldots \quad (1)$$

The phase angle of the load is given by

$$\tan\theta = \frac{\omega\{L(1-\omega^2 LC) - CR^2\}}{R}$$

and the frequency of the radio frequency oscillation is accordingly given by $\tan\theta = -\sqrt{3}$ (Rakshit and Bhattacharyya, 1946)

Or
$$\omega^3 L^2 C + \omega(CR^2 - L) - \sqrt{3}R = 0 \qquad \ldots \quad (2)$$

This is a cubic equation with one real root, the other two being imaginary. The real value of ω is given by

$$\omega = [P + (P^2 + Q^3)^{\frac{1}{2}}]^{1/3} + [P - (P^2 + Q^3)^{\frac{1}{2}}]^{1/3} \qquad \ldots \quad (3)$$

where

$$P = \frac{\sqrt{3}R}{2L^2 C} \qquad \text{and} \qquad Q = \frac{CR^2 - L}{3L^2 C}$$

For convenience of numerical computation, this may be written in the following form by putting x for CR^2/L. We then have

$$\omega = \frac{x^{2/3}}{CR}\left[\left\{\frac{\sqrt{3}}{2} + \left(0.75 + \frac{(x-1)^3}{27x}\right)^{\frac{1}{2}}\right\}^{1/3} + \left\{\frac{\sqrt{3}}{2} - \left(0.75 + \frac{(x-1)^3}{27x}\right)^{\frac{1}{2}}\right\}^{1/3}\right] \qquad \ldots \quad (3a)$$

$$= \frac{K}{CR}, \qquad \ldots \quad (3b)$$

where $K = x^{2/3}\left[\left\{\frac{\sqrt{3}}{2}+\left(0.75+\frac{(x-1)^3}{27x}\right)^{\frac{1}{2}}\right\}^{1/3}\right.$

$$\left.+\left\{\frac{\sqrt{3}}{2}-\left(0.75+\frac{(x-1)^3}{27x}\right)^{\frac{1}{2}}\right\}^{1/3}\right].$$

It is obvious that if a table giving K for different values of x according to this equation is available we can at once calculate ω from Eqn. (3b) by first finding x for the particular combination of R, L and C. For this purpose the calculated values of K for different values of x ranging from 0.01 to 100 have been given in Table 1 The results are also plotted in Fig. 1. For values of $x > 100$ or $<$

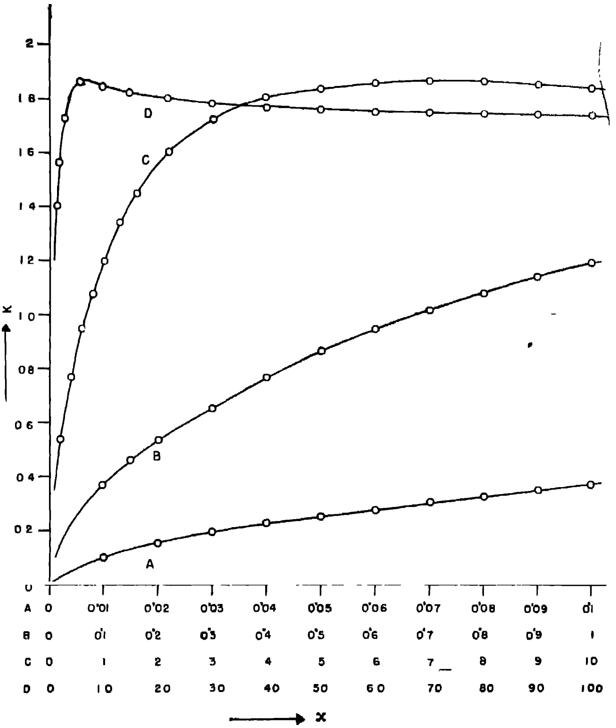

Fig. 1. Variation of K with x.

<0.01 we can calculate the value of ω after making some approximations leading to Eqns. (4a) and (7a). The error involved in these approximations is quite small. Thus for $x = 100$ if ω is calculated by Eqn. (4a) instead of Eqn. (3a), the result is 0.117% greater than the exact value. For larger values of x the error due to approximation will gradually diminish. Similarly when $x = 0.01$ if ω is calculated by Eqn (7a) the result is 1.2% greater than the exact value. For still smaller values of x the error due to approximation is still less.

TABLE 1

x *vs.* K

x	K	x	K	x	K
0.01	0.1073	1	1.2010	6	1.8633
0.02	0.1547	1.1	1.2486	7	1.8661
0.03	0.1930	1.2	1.2971	8	1.8637
0.04	0.2251	1.3	1.3389	9	1.8551
0.05	0.2540	1.4	1.3789	10	1.8537
0.06	0.2805	1.5	1.4154	11	1.8479
0.07	0.3049	1.6	1.4492	12	1.8416
0.08	0.3272	1.7	1.4804	13	1.8363
0.09	0.3495	1.8	1.5134	15	1.8263
0.1	0.3699	1.9	1.5363	22	1.8014
0.15	0.4606	2	1.5613	25	1.7945
0.20	0.5375	2.2	1.6062	30	1.7854
0.30	0.6556	2.5	1.6622	40	1.7735
0.40	0.7724	2.8	1.7108	50	1.7648
0.50	0.8660	3	1.7320	60	1.7559
0.60	0.9477	3.4	1.7718	70	1.7547
0.70	1.0217	3.7	1.7949	80	1.7534
0.80	1.0859	4	1.8137	90	1.7520
0.90	1.1445	5	1.8488	100	1.7472

It will be noted that when Q is negative and $P^2 < |Q^3|$, $\{P^2+Q^3\}^{\frac{1}{2}}$ is imaginary and Eqn. (3) can be solved with the help of trigonometry but not algebraically.

The expression for ω as given in Eqn.(3) can be reduced to a simpler approximate form when one of the terms under the square root becomes small in comparison with the other. Thus when $CR^2 > L$, i.e. Q is positive and $P^2 << Q^3$

i.e. $\frac{81CR^2}{4L} \qquad \left(\frac{CR^2}{L} - 1\right)^3$, we get (see Appendix I)

$$\omega = \frac{\sqrt{3R}}{CR^2 - L}\left[1 - \frac{3R^2L^2C}{(CR^2-L)^3}\right] \qquad \ldots \quad (4)$$

Or

$$\omega = \frac{\sqrt{3x}}{CR(x-1)}\left[1 - \frac{3x}{(x-1)^3}\right] \qquad \ldots \quad (4a)$$

As a further approximation when $CR^2 >> L$ we may neglect the term $\frac{3R^2L^2C}{(CR^2-L)^3}$ and Eqn (4) reduces to

$$\omega = \frac{\sqrt{3}R}{CR^2-L} \qquad \ldots \quad (5)$$

This shows that when L is so small that $P^2 << Q^3$ the small inductance associated with the anode load resistance causes an increase in frequency. In the limit when $L = 0$, we get the simple expression $\omega = \sqrt{3}/CR$

When Q is negative and $P^2 < |Q^3|$, solving Eqn. (3) trigonometrically gives

$$\omega = r^{1/3}\, 2\cos\frac{\phi}{3} \qquad \ldots \quad (6)$$

where $r = |Q^{3/2}|$ and $\cos\phi = P/Q^{3/2}$. Substituting the values of P and Q,

$$\omega = \left|\frac{1}{3LC}\left(\frac{CR^2}{L}-1\right)\right|^{\frac{1}{2}} 2\cos\frac{\cos^{-1}\left|\dfrac{81CR^2}{4L\left(\dfrac{CR^2}{L}-1\right)^3}\right|}{3} \qquad \ldots \quad (6a)$$

When R is very small Eqn (6) reduces to (see Appendix II)

$$\omega = \frac{1}{\sqrt{LC}} + \frac{\sqrt{3}R}{2L} \qquad \ldots \quad (7)$$

Or in terms of x,

$$\omega = \frac{\sqrt{x}}{CR}\left(1+\frac{\sqrt{3x}}{2}\right) = \frac{1}{\sqrt{LC}}\left(1+\frac{\sqrt{3x}}{2}\right) \qquad \ldots \quad (7a)$$

This shows that when R is of the order of 0 the frequency generated by a 3-phase oscillator is approximately equal to the frequency generated by an orthodox $L-C$ oscillator with same circuit parameters but always greater than $1/\sqrt{LC}$. The expression $\omega = 1/\sqrt{LC}$ in case of 3-phase oscillator can also be obtained by putting $R = 0$ in Eqn. (2) but from the expression for $\tan\theta$ it is obvious that oscillations cannot be maintained when $R = 0$.

The above Eqn. (6) holds good in the case $CR^2 < L$ upto the limit $P^2 = |Q^3|$. Under this limiting condition $\cos\phi = 1$ and hence $\cos\frac{\phi}{3} = 1$ and

$$\omega = 2\left\{\frac{\sqrt{3}R}{2L^2C}\right\}^{1/3} \qquad \ldots \quad (8)$$

It will be noted that when the above condition holds, $P^2+Q^3 = 0$ and the same expression for ω is also obtained directly from Eqn. (3).

When, on the other hand, $P^2 >> Q^3$

$[P + (P^2 + Q^3)^{\frac{1}{2}}]^{1/3}$ is of the order of $(2P)^{1/3} + \frac{1}{12}\frac{Q^3}{P^2}(2P)^{1/3}$ and $[P-(P^2+Q^3)^{\frac{1}{2}}]^{1/3}$ is of the order of $-\left(\frac{Q}{2P}\right)^{1/3}$.

$$\because \quad \omega = (2P)^{1/3} - \left(\frac{Q}{2P}\right)^{1/3} + \frac{1}{12}\frac{Q^3}{P^2}(2P)^{1/3} \qquad \ldots \quad (9)$$

Or, in terms of x,

$$\omega = \frac{1}{CR}\left[(\sqrt{3}x^2)^{1/3} - \frac{x^{1/3}(x-1)}{3^{7/6}} + \frac{(x-1)^3}{x^{1/3}3^{29/6}}\right] \qquad \ldots \quad (9a)$$

For generating very high frequency the magnitudes of R, C and L should be very small; (CR^2-L) is then usually a very small quantity In the special case when $CR^2-L = 0$ i.e. $x = 1$, Eqn. (9a) reduces to

$$\omega = \frac{3^{1/6}}{CR} = \frac{1.201}{CR} \qquad \ldots \quad (10)$$

Condition for Maintenance of Oscillation

We have so far considered the phase shift per stage necessary to produce oscillations. For maintenance of these oscillations the gain A per stage must at least be unity. If anode load impedance of each oscillator stage is Z_0 as given in Eqn. (1). this means

$$g_m Z_0 = A \geqslant 1$$

where g_m is the mutual conductance of the oscillator valves. To evaluate Z_0 from Eqn (1) in terms of circuit parameters we have to substitute the value of ω in terms of these parameters Taking ω as given by Eqn. (3b), we find

$$Z_0 = \frac{R\left[1 + \frac{K^2}{x^2}\left\{1 - \frac{K^2}{x} - x\right\}^2\right]^{\frac{1}{2}}}{K^2 + \left(\frac{K^2}{x} - 1\right)^2}$$

For a particular combination of C,R and L, since x and K are pure numbers Z_0 is obtained in terms of R only. It is easy to see that Z_0 can likewise be expressed in terms of any one of the circuit parameters. The magnitude of Z_0 in terms of R for various values of x and the condition that has to be satisfied to maintain oscillation in those cases are given in Table II and plotted in figure 2. It may be noted that when $L = 0$ we have the simple $R-C$ oscillator and $\omega = \sqrt{3}/CR$,

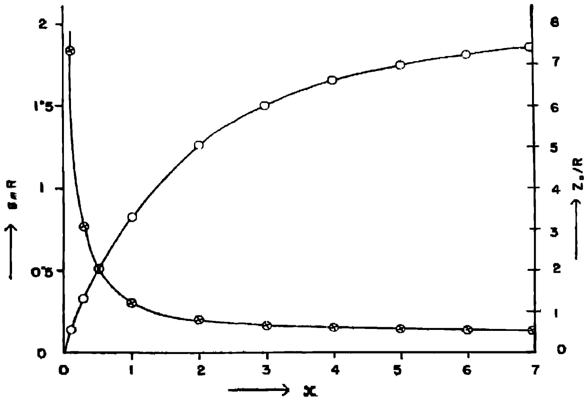

Fig. 2. Z_0 in terms of R, and g_m R required for maintenance of oscillation for different values of x.

TABLE II

Z_0 *vs* x and minimum g_m R for oscillation

x	Z_0	Min. g_mR	Remarks
∞	0.5 R	2	Purely resistive load
6.964	0.53 R	1.866	$\frac{\partial\omega}{\partial L}=0$
6	0.546 R	1.813	
5	0.568 R	1.759	
4	0 603 R	1.659	
3	0.666 R	1.50	$\omega_{RC}=\omega_{RLC}$
2	0.792 R	1 263	
1	1.221 R	0.818	$CR^2=L$
0.5	2 R	0 5	$\frac{\partial\omega}{\partial R}=0$
0.3	3.06 R	0.327	
0.1	7.342 R	0.136	

$Z_0 = R/2$ and condition for maintenance is $g_m R \gg 2$. When $R = 0$, ω is of the order of $1/\sqrt{LC}$ (Eqn. 7), Z_0 is very large and oscillation can be maintained with only a very small value of g_m.

NATURE OF VARIATION OF ω WITH THE TUNING PARAMETERS IN THE THREE-PHASE OSCILLATOR

The equations deduced above give the variations of ω when the tuning parameters are changed. Some important results are obtained if we study the variation with any one parameter at a time.

(*a*) *Variation of ω with R*

1. In the case of the $R-C$ oscillator the frequency is given by $\omega = \sqrt{3}/CR$.

$$\frac{\partial\omega}{\partial R} = -\frac{\sqrt{3}}{CR^2} \qquad \ldots \quad (11)$$

and is always negative.

2. In the case of the $R-L-C$ oscillator the real value of ω is given by Eqn. (3), but it is rather difficult to find $\partial\omega/\partial R$ from this expression for ω. It is more convenient to find $\partial\omega/\partial R$ starting from Eqn. (2) Thus if we put

$$\omega^3 L^2 C + \omega(CR^2 - L) - \sqrt{3}R = F$$

then

$$\frac{\partial\omega}{\partial R} = -\frac{\partial F}{\partial R}\bigg/\frac{\partial F}{\partial\omega} = -\frac{2\omega CR - \sqrt{3}}{3\omega^2 L^2 C + (CR^2 - L)} \qquad \ldots \quad (12)$$

For any particular set of values for R, L and C, we find ω from Eqn. (3) and substituting this value of ω in Eqn. (12) we get $\partial\omega/\partial R$. Now from Eqn (2), $\omega^2 L^2 C + (CR^2 - L) = \frac{\sqrt{3}R}{\omega}$ and hence Eqn. (12) may be written as

$$\frac{\partial\omega}{\partial R} = \frac{\sqrt{3} - 2\omega CR}{2\omega^2 L^2 C + \frac{\sqrt{3}R}{\omega}} \qquad \ldots \quad (12a)$$

The denominator being always positive, we see that the nature of variation of ω is dependent upon whether $2\omega CR$ is greater or less than $\sqrt{3}$. In the limit when $L = 0$, we have the simple $R-C$ oscillator and $\omega = \sqrt{3}/CR$ and Eqn. (12a) gives

$$\frac{\partial\omega}{\partial R} = -\frac{\sqrt{3}}{\sqrt{3}R/\omega} = -\frac{\omega}{R} = -\frac{\sqrt{3}}{CR^2}$$

exactly as in Eqn. (11a). Equation (12a) further shows that if the set of values of R, L and C is such that $\sqrt{3}/2CR > \omega$, $\partial\omega/\partial R$ is positive for the $R-L-C$ oscillator and not negative as is always the case with simple $R-C$ oscillator. In fact for very small values of R, $\partial\omega/\partial R$ is always positive, since $2\,CR\omega$ approaches zero as R becomes negligibly small.

Further if R is gradually increased, keeping the above magnitudes of L and C constant, we come to a value of $\omega = \sqrt{3}/2CR$ for which $\partial\omega/\partial R = 0$. This maximum value of ω for which $\partial\omega/\partial R = 0$ may be calculated as follows:
We have $\omega_{max} = \sqrt{3}/2CR_{max}$, where R_{max} is the value of R for which $\partial\omega/\partial R = 0$. Substituting this value of R in Eqn (2), we have

$$\omega^3L^2C - \frac{3}{4\omega C} - \omega L = 0$$

or

$$4\omega^4L^2C^2 - 4\omega^2LC - 3 = 0$$

Solving as a quadratic in $2\,\omega^2LC$, we get

$$2\omega^2LC = 3 \text{ or } -1,$$

the negative value being obviously inadmissible

$$\omega_{max} = \left[\frac{3}{2LC}\right]^{\frac{1}{2}} \quad \ldots \quad (13a)$$

and

$$R_{max} = \left[\frac{L}{2C}\right]^{\frac{1}{2}} \quad \ldots \quad (13b)$$

These equations show that if the ratio L/C is kept constant, the maximum frequency for which $\partial\omega/\partial R = 0$ will correspond to a fixed value of resistance irrespective of the individual magnitudes of L and C. The magnitude of the maximum frequency which depends on the product LC will however be different for different magnitudes of L and C.

For further increase in R, $\delta\omega/\partial R$ is negative and at a certain value of R the frequency of the $R-L-C$ oscillator once again becomes equal to $\sqrt{3}/CR$ as if L were not present. This value of R for which the presence or absence of L makes no difference in the frequency of the oscillations may be obtained by putting $\omega = \sqrt{3}/CR$ in Eqn. (2). This gives

$$\frac{3\sqrt{3}L^2C}{C^3R^3} + \frac{\sqrt{3}CR^2}{CR} - \frac{\sqrt{3}L}{CR} - \sqrt{3}R = 0$$

or

$$\frac{\sqrt{3}L}{CR}\left(\frac{3L}{CR^2} - 1\right) = 0$$

which is satisfied by

$$\text{(i) } L = 0 \text{ and (ii) } CR^2 = 3L \quad \ldots \quad (14)$$

Condition (i) is obvious and condition (ii) gives the desired value of R.

It is interesting to compare the value of $\partial\omega/\partial R$ at the same value of ω, for the two cases —(i) $R-C$ oscillator and (ii) $R-L-C$ oscillator with $R=[3L/C]^{\frac{1}{2}}$ i.e., $CR^2=3L$. For case (i), $\partial\omega/\partial R=-\sqrt{3}/CR^2$ as already seen. For case (ii), substituting $\omega=\sqrt{3}/CR$ and $L=CR^2/3$ in Eqn (12) gives

$$\left(\frac{\partial\omega}{\partial R}\right)_{RLC}=-\frac{\sqrt{3}}{\frac{5}{3}CR^2}=\frac{3}{5}\left(\frac{\partial\omega}{\partial R}\right)_{RC} \tag{15}$$

This shows that for the same value of ω with a particular value of R and C the stability of frequency with respect to variation in R is higher if an inductance $L=CR^2/3$ is included in series with the anode load resistance.

To minimise this type of frequency variation it is obviously best to work in the region where $\partial\omega/\partial R=0$. Since we have two Eqns (13a and 13b) and three quantities R_{max}, L and C to be estimated for the desired value of ω_{max} to be maintained, we have a wide choice in the selection of these parameters. From the point of view of reduction in frequency variation due to changes in valve inter-electrode capacitances we should select as large a value of C as is possible.

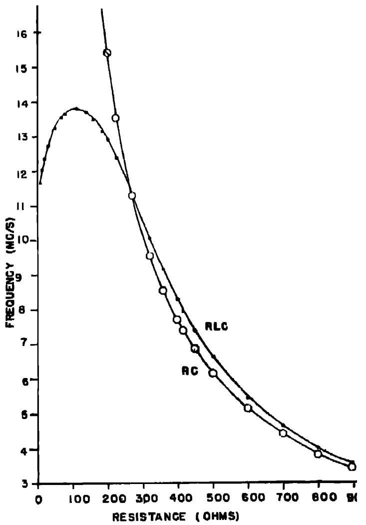

Fig 3. Variation of Frequency with Resistance.

Thus if the desired frequency is 20 Mc/s. and is to be obtained with $C=100\mu\mu F$, Eqn. (13a) gives L is of the order of $0.964\mu H$ and Eqn. (13b) gives R_{max} is of the order of 68.94 ohms.

The frequencies calculated according to Eqn. (3) for different values of R and (i) with fixed values of L and C for $R-L-C$ oscillator and (ii) fixed value of C for $R-C$ oscillator are given in Table III and the variations of frequency with R are plotted in Fig 3. The curves in Fig. 3 further show that for the typical values of L and C chosen in the example although $\partial\omega/\partial R$ is negative in either case for values of $R > [L/2C]^{\frac{1}{2}}$, the magnitude of $\partial\omega/\partial R$ is lower for the RLC case for moderate values of R. As R is further increased $\partial\omega/\partial R$ becomes equal for the two cases and for still larger values of R, $\partial\omega/\partial R$ for the RLC case becomes higher than that for the RC case Again for very high resistances the slopes tend towards equality

TABLE III

Variation of frequency with R

$C = 89.4\mu\mu F$; $L = 2.2277\mu H$

R in ohms (Effective value)	Frequency in Mc/s		$f_{R-L-C} - f_{R-C}$ in Mc/s
	$R-L-C$	$R-C$	
5	11.672	Oscillation is not possible with valves having g_m not greater than 10_mA/V	
13.5	12.02		
22	12.357		
32	12.717		
52	13.237		
70	13.55		
80	13.663		
100	13.802		
111.6	13.812		
138	13.723		
160	13.505		
185	13.413		
200	12.91	15.417	−2.507
228	12.348	13.520	1.172
273.5	11.27	11.27	0
323	10.044	9.562	+0.482
360	9.159	8.565	0.594
400	8.296	7.709	0.587
416	7.930	7.403	0.527
450	7.370	6.852	0.518
500	6.613	6.617	0.446
600	5.427	5.139	0.302
700	4.602	4.404	0.198
800	3.994	3.854	0.140
900	3.527	3.429	0.098
1000	3.155	3.083	0.072

(*b*) *Variation of* ω *with* L.

This case applies only to the $R-L-C$ oscillator and proceeding as already discussed we get

$$\frac{\partial\omega}{\partial L}=-\frac{\partial F}{\partial L}\bigg/\frac{\partial F}{\partial\omega}=\frac{\omega(1-2LC\omega^2)}{3\omega^2L^2C+(CR^2-L)}=\frac{\omega(1-2LC\omega^2)}{2\omega^2L^2C+\frac{\sqrt{3}R}{\omega}}\quad\ldots\quad(16)$$

Here also, as in case of variation of ω with R, the nature of variation of ω is dependent upon whether $2LC\omega^2$ is greater or less than unity. For negligibly small values of L, $\partial\omega/\partial L$ is positive and with increase of L the rate is gradually reduced to zero and thereafter $\partial\omega/\partial L$ becomes negative. The maximum value of ω when $\partial\omega/\partial L=0$ is given by $\omega_{max}=1/[2CL_{max}]^{\frac{1}{2}}$, where L_{max} is the value of L for which $\partial\omega/\partial L=0$. Substituting this value of L_{max} in Eqn. (2), we get

$$4\omega^2C^2R^2-4\sqrt{3}\omega CR-1=0$$

$\therefore\quad \omega_{max}=(\sqrt{3}\pm 2)/2CR$. The negative sign is obviously inadmissible and hence

$$\omega_{max}=1.866/CR \text{ and } L_{max}=0.1435\,CR^2 \quad\ldots\quad(17)$$

After reaching maximum frequency for $L=L_{max}$, $\partial\omega/\partial L$ becomes negative with further increase in L. At a certain value of L greater than L_{max} the fre-

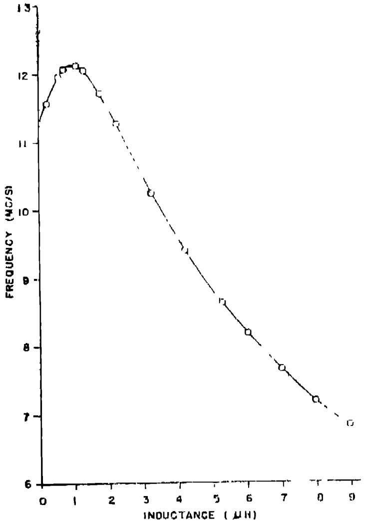

Fig. 4. Variation of frequency with inductance.

quency becomes equal to that of a simple $R-C$ oscillator. This value of L is, as before, given by $CR^2=3L$.

The frequencies calculated according to Eqn. (3) for different values of L and with fixed values of R and C are given in Table IV and the results are plotted in Fig. 4.

TABLE IV

Variation of frequency with L

$C = 89.4\mu\mu F$; $R = 273.5\ \Omega$

Inductance in μH	Frequency in Mc/s.	Inductance in μH.	Frequency in Mc/s.
0.1937	11.580	6	8 1713
0.5444	11.979	7	7 665
0 7087	12.085	8	7.2042
0.9693	12.147	9	6 834
1.062	12 136	11 00	6.216
1 30	12 06	13.374	5.6063
1 749	11.725	15 00	5.3179
2.2277	11.27	17 00	4.979
3.2386	10.26	20.00	4.5958
4.1824	9.4155	30 00	3.677
5.2923	8.6261	40.00	3 1776

(c) *Variation of* ω *with* C.

1. In case of $R-C$ oscillator $\omega = \sqrt{3}/CR$ and

$$\frac{\partial\omega}{\partial C} = -\frac{\sqrt{3}}{C^2R} \qquad \dots \quad (18)$$

and is always negative.

2. In case of the $R-L-C$ oscillator,

$$\frac{\partial\omega}{\partial C} = -\frac{\partial F}{\partial C}\Big/\frac{\partial F}{\partial\omega} = -\frac{\omega^3L^2+\omega R^2}{2\omega^2L^2C+\dfrac{\sqrt{3}R}{\omega}} \qquad \dots \quad (19)$$

and is always negative. When C is negligibly small, Eqn. (2) reduces to $\omega L + \sqrt{3}R = 0$, i.e. oscillations are not possible with very small values of C. At a value of $C = 3L/R^2$, $\omega_{LC} = \omega_{RC} = \sqrt{3}/CR = R/\sqrt{3}L$, and substituting $\omega = \sqrt{3}/CR$ and $L = CR^2/3$ in Eqn. (19) we get

$$\frac{\partial\omega}{\partial C} = -\frac{4}{5}\frac{\sqrt{3}}{C^2R} = \frac{4}{5}\left(\frac{\partial\omega}{\partial c}\right)_{RC} \qquad \dots \quad (20)$$

This shows that with a particular combination of R and C, the stability of frequency with respect to variation in C is higher if an inductance $L = CR^2/3$ is included in series with the anode load resistance. Equations (15) and (20) show that this inductance improves frequency stability with regard to variations in R as well as C.

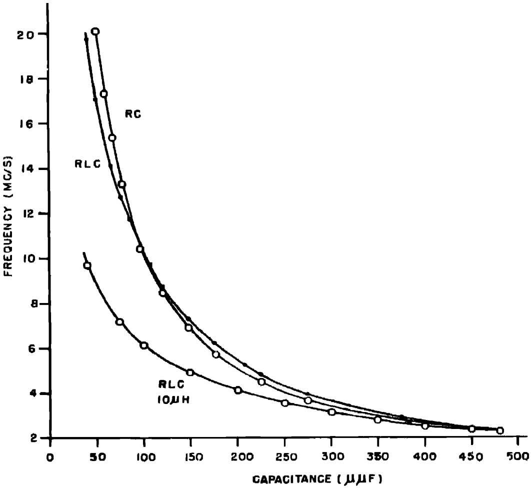

Fig. 5. Variation of frequency with capacitance

The frequencies calculated according to Eqn. (3) for different values of C and (i) with fixed values of L and R for $R-L-C$ oscillator and (ii) with fixed value of R for the $R-C$ oscillator are given in Table V and the results are plotted in Fig. 5. In the same figure the variation of frequency with capacity for same R but $L = 10\mu H$ is also plotted.

EFFECT OF CATHODE CIRCUIT IMPEDANCE ON OSCILLATOR FREQUENCY

The impedance of the cathode circuit of an oscillator valve may be resistive, inductive or capacitive depending on the magnitude of cathode biasing capacitance and operating frequency. The nature of impedance of the cathode circuit at different frequencies with different circuit components has been discussed in Part I in connection with $R-C$ oscillator with non-inductive load resistance. It has been pointed out there that the effect of cathode impedance Z_k on the oscil-

TABLE V

Variatiln of frequency with C
$R = 273.50\Omega$; $L = 2.2277\mu H$

Capacitance in $\mu\mu F$	Frequency in Mc/s		Capacitance in $\mu\mu F$.	Frequency in Mc/s.	
	R–L–C	R–C		R–L-C	R–C
40	19.72	25.19	120	8.743	8.420
49.5	17.05	20.04	133	7.975	7.578
58	15.355	17.35	147	7.276	6.856
66	14.039	15.27	175	6.184	5.772
76	12.665	13.26	207.4	5.236	4.860
85	11.633	11.857	225	4.821	4.478
89.4	11.27	11.27	275	3.937	3.664
96	10.556	10.499	375	2.845	2.694
106	9.724	9.509	480	2.209	2.120

lator load is to change its effective value When, for example, the screen is decoupled to ground the anode load Z_0 is changed to (Sturley, 1949)

$$Z_{eff} = \frac{Z_0}{1+g_k Z_k} \qquad \dots \quad (21)$$

where $g_k = g_m + g_s$, g_s being screen current-grid voltage slope conductance.

The presence of some impedance in the cathode circuit also influences the input admittance of the valve which becomes perceptible only at high frequency. In the present case the effect of change in input admittance on the oscillator frequency which is very small, has been ignored.

Substituting the value of Z_0 from Eqn. (1) we find, on simplification, that the phase angle θ of the effective load is given by

$$\tan\theta = \frac{\omega\{L - \omega^2 L^2 C - CR^2\}\{1 + g_k r\} - g_k XR}{R(1 + g_k r) + \omega X g_k (L - \omega^2 L^2 C - CR^2)} \qquad (22)$$

where r and X are the resistive and reactive parts of the cathode impedance. The frequency of the oscillation is given by $\tan\theta = -\sqrt{3}$

Or

$$-\sqrt{3} = \frac{\{\omega L - \omega^3 L^2 C - \omega CR^2\}\{1 + g_k r\} - g_k XR}{R(1+g_k r)+\omega X g_k(L-\omega^2 L^2 C - CR^2)} \qquad (23)$$

Near about the resonant frequency of the cathode circuit the magnitude of the reactive component X is negligible and then the frequency of oscillation is given by

$$-\sqrt{3} = \frac{\{\omega L - \omega^3 L^2 C - \omega C R^2\}\{1+g_k r\}}{R(1+g_k r)}$$

or

$$\omega^3 L^2 C + \omega(CR^2 - L) - \sqrt{3}R = 0$$

which, as expected, is the same as Eqn. (2).

When the generated frequency is greater than the resonant frequency of the cathode circuit the cathode impedance will be inductive and we may put $X = \omega_1 L'$, where

L' = effective inductance of the cathode circuit which is also a function of the generated frequency ω_1.

Equation (23) then reduces to

$$\sqrt{3}\omega_1^4 L^2 C L' g_k + \omega_1^3 L^2 C(1+g_k r) + \omega_1(CR^2 - L)(1+g_k r)$$
$$-\sqrt{3}R(1+g_k r) + \sqrt{3}\omega_1^2 L' g_k (CR^2 - L) + \omega_1 L' g_k R = 0 \qquad \ldots \quad (24)$$

This being a bi-quadratic equation in ω_1 it is not possible to have a simple solution. It can however be shown that when the cathode circuit is inductive the generated frequency decreases. This can be seen by comparing Eqn. (24) which may be put in the form

$$\omega_1^3 L^2 C + \omega_1(CR^2 - L) - \sqrt{3}R = -\frac{4\omega_1 L' g_k R}{1+g_k r + \sqrt{3}\omega_1 L' g_k} \qquad (24a)$$

and Eqn (2) according to which the generated frequency is ω when the cathode circuit is resonant, given by

$$\omega^3 L^2 C + \omega(CR^2 - L) - \sqrt{3}R = 0$$

Since the right hand side of Eqn. (24a) is essentially a negative quantity, $\omega_1 < \omega$ because the anode circuit parameters L, C and R are maintained constant.

The absolute difference between ω and ω_1 naturally depends upon the magnitude of $4\omega_1 L' g_k R/(1+g_k r+\sqrt{3}\omega_1 L' g_k)$. When either g_k or R or both are small $\omega - \omega_1$ is also small, i.e the influence of cathode circuit on the oscillator frequency will be small.

Similarly when the cathode circuit is capacitive i.e. the generated frequency is less than the resonant frequency of the cathode circuit, we may put $X = -1/\omega_2 C'$ where C' = effective capacitance of the cathode circuit which is of course a function of the generated frequency ω_2. Equation 23 then reduces to

$$\omega_2^3 L^2 C + \omega_2(CR^2 - L) - \sqrt{3}R = \frac{4g_k R}{\omega_2 C''(1+g_k r) - \sqrt{3}g_k} \quad \dots \quad (25)$$

The right hand side of Eqn. (25) may be positive or negative depending mainly on the magnitude of C''. It may be mentioned here that in any practical case C'' never reaches such a low value as to make $\omega_2 C''(1+g_k r)$ less than $\sqrt{3}g_k$. The right hand side of Eqn. (25) will then be positive. Now comparing Eqn. (25) with Eqn. (2) we at once see that $\omega_2 > \omega$, since the anode circuit parameters L, C and R are constant. The absolute difference between ω_2 and ω being dependent on the magnitude of $4g_k R/\{\omega_2 C''(1+g_k r) - \sqrt{3}g_k\}$, when either g_k or R or both are small, $\omega_2 - \omega$ is also small, i.e. the influence of cathode circuit on oscillator frequency will be small.

CONCLUSION

The effect of an added inductance in series with the load resistance of the three-phase oscillator, and in such case the effect of cathode impedance, on the frequency of oscillation has been discussed in detail in the present paper. An interesting feature which has been observed is that $\partial\omega/\partial R$ and $\partial\omega/\partial L$ may have both positive and negative values. These features can be utilized to eliminate frequency variation of the oscillator due to change in temperature. The effect of cathode impedance on the oscillator frequency is very small unless the cathode circuit resonant frequency is far off from the generated frequency and Q factor of the anode circuit is very low. The conclusions arrived at different stages of this paper have been verified experimentally. The results of such observations and detailed design procedure for better frequency stability of the oscillator are being communicated separately.

APPENDIX I

$$\omega = [P+(P^2+Q^3)^{\frac{1}{2}}]^{1/3} + [P-(P^2+Q^3)^{\frac{1}{2}}]^{1/3}$$

where $$P = \frac{\sqrt{3}R}{2L^2C} \quad \text{and} \quad Q = \frac{CR^2 - L}{3L^2C}.$$

When Q is positive and $P^2 << Q^3$,

$$[P+(P^2+Q^3)^{\frac{1}{2}}]^{1/3}$$

$$= \left[P + Q^{3/2}\left\{1 + \frac{1}{2}\frac{P^2}{Q^3} - \frac{1}{8}\frac{P^4}{Q^6} + \dots \right\}\right]^{1/3}$$

$$= Q^{\frac{1}{2}}\left[1 + \frac{P}{Q^{3/2}} + \frac{1}{2}\frac{P^2}{Q^3} - \frac{1}{8}\frac{P^4}{Q^6} + \dots \right]^{1/3},$$

neglecting other terms which are insignificant in magnitude.

$$= Q^{\frac{1}{2}}\left[1+\frac{1}{3}\left(\frac{P}{Q^{3/2}}+\frac{1}{2}\frac{P^2}{Q^3}-\frac{1}{8}\frac{P^4}{Q^6}\right)-\frac{1}{9}\left(\frac{P}{Q^{3/2}}+\frac{1}{2}\frac{P^2}{Q^3}-\frac{1}{8}\frac{P^4}{Q^6}\right)^2 + \frac{5}{81}\left(\frac{P}{Q^{3/2}}+\frac{1}{2}\frac{P^2}{Q^3}-\frac{1}{8}\frac{P^4}{Q^6}\right)^3 \cdots\right]$$

is of the order of $Q^{\frac{1}{2}}\left[1+\frac{1}{3}\frac{P}{Q^{3/2}}+\frac{1}{18}\frac{P^2}{Q^3}+\frac{4}{81}\frac{P^3}{Q^{9/2}}+\frac{5}{216}\frac{P^4}{Q^6}\right]$.

Similarly $\quad [P-(P^2+Q^3)^{\frac{1}{2}}]^{1/3}$

is of the order of $-Q^{\frac{1}{2}}\left[1-\frac{1}{3}\frac{P}{Q^{3/2}}+\frac{1}{18}\frac{P^2}{Q^3}+\frac{4}{81}\frac{P^3}{Q^{9/2}}+\frac{5}{216}\frac{P^4}{Q^6}\right]$

$$\therefore \quad \omega = Q^{\frac{1}{2}}\left[\frac{2}{3}\frac{P}{Q^{3/2}}-\frac{8}{81}\frac{P^3}{Q^{9/2}}\right] \text{ approximately}$$

$$= \frac{2}{3}\frac{P}{Q}\left(1-\frac{4}{27}\frac{P^2}{Q^3}\right)$$

Substituting the values of P and Q

$$\omega = \frac{\sqrt{3}R}{CR^2-L}\left[1-\frac{3R^2L^2C}{(CR^2-L)^3}\right]$$

APPENDIX II

When R is very small, $CR^2 << L$ and $P^2 << Q^3$

Under these conditions $\cos\phi = \frac{P}{Q^{3/2}} = 0$

and we may put $\phi = \frac{\pi}{2} - \beta$, where β is very small

Then $\quad \cos\frac{\phi}{3} = \cos\left(30^\circ - \frac{\beta}{3}\right)$ is of the order of $\frac{\sqrt{3}}{2}+\frac{1}{2}\frac{\beta}{3}$

Now $Q = -\frac{1}{3LC}\left(1-\frac{CR^2}{L}\right)$ and $\left|Q^{3/2}\right|$ is of the order of $\frac{1}{(3LC)^{3/2}}\left(1-\frac{3}{2}\frac{CR^2}{L}\right)$

$$\therefore \left|\frac{P}{Q^{3/2}}\right| = \frac{\sqrt{3}R}{2L^2C}\cdot\frac{(3LC)^{3/2}}{1-\frac{3}{2}\frac{CR^2}{L}} \text{ is of the order of } \frac{9R}{2}\sqrt{\frac{C}{L}}\left(1+\frac{3}{2}\frac{CR^2}{L}\right).$$

Neglecting $\frac{3}{2}\frac{CR^2}{L}$ which is very small in comparison to 1,

$$\cos\phi = \sin\beta = \frac{9R}{2}\sqrt{\frac{C}{L}} \quad \text{or} \quad \frac{\beta}{3} = \frac{3R}{2}\,\frac{L}{C}$$

and hence $$\cos\frac{\phi}{3} = \frac{\sqrt{3}}{2} + \frac{1}{2}\,\frac{3R}{2}\sqrt{\frac{C}{L}} = \frac{1}{2}\left\{\sqrt{3} + \frac{3R}{2}\sqrt{\frac{C}{L}}\right\}$$

Substituting this value of $\cos\phi/3$ in Eqn. (6)

$$\omega = \frac{1}{\sqrt{LC}} + \frac{\sqrt{3}R}{2L}$$

REFERENCES

Rakshit, H. and Bhattacharyya, K. K., 1946, *Ind. J. Phys*, **20**, 171.
Rakshit, H. and Mallik, M. C., 1953, *Jour. Sci. & Ind. Res.*, **12B**, 30.
Rakshit, H. and Mallik, M. C., 1955, *Ind. J. Phys.*, **30**, 534.
Sturley, K. R., 1949, Radio Receiver Design Part One. (Chapman & Hall, London).